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Methods to increase sound fidelity and quality produced from piezoelectric devices

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Abstract

Methods to increase sound level, fidelity and quality produced from vibrating lamina such as piezoelectric actuators, vibrating plates or vibrating films are presented. Results of using the methods are shown for piezoelectric devices. Four methods are described: (1) tailoring the vibration response to develop desired deformation shapes and amplitudes, (2) mapping vibration out-of-plane deformation to ascertain locations on the surfaces of lamina suitable for stroke-like actuation, (3) coupling vibration to a collection of acoustic chambers and (4) increasing the vibration decay rate. The first two methods provide a single piezoelectric device with the functionality of numerous actuators combined. A piezoelectric actuator with numerous high-amplitude natural vibration responses has been produced using the aforementioned methods. Numerous high-amplitude vibrations increase the functionality of the devices. A collection of acoustic chambers were affixed to the piezoelectric actuator's surface. The addition of the chambers resulted in more efficient audio output. The result of using all of the aforementioned methods is a high fidelity, high-bandwidth, and high sound-quality audio device with a low physical profile. The use of the piezoelectric actuator effectively results in an audio driver with a thickness less than 1 mm. The piezoelectric audio device achieved a response of 93 ± 4 dB measured at 1 cm in the frequency range (1–5 kHz) with very good audio output for frequencies less than 1 kHz. The methods can be used to design other devices using solid-state piezoelectric actuators or vibrating lamina (e.g., plates or films).

1. Introduction

This monograph presents methods of harnessing piezoelectric vibration to produce a low-profile audio device with high-bandwidth high-fidelity response. Fundamentals of piezoelectric devices can be found in Ref. [1]. One method consists of tailoring dynamic attributes of the

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piezoelectric actuator to produce a vibration response with numerous high-amplitude modes. The tailored attributes include mass distribution, stiffness distribution, damping distribution and boundary conditions. The vibration deformation topography was mapped for all natural resonances to determine the most effective locations for force actuation for all vibration modes within the frequency range of interest. For any natural frequency, there will be one or more positions on the actuator surface that could be used for stroke-like actuation. In essence, a piezoelectric device that previously functioned as a single actuator could now function as a collection of actuators.

Numerous Helmholtz-like acoustic chambers affixed to the surface of the device provided a very easy means of demonstrating the aforementioned thesis. The chambers were located at the locations upon the piezoelectric device surface where the vibration deformation peak out-of-plane displacements coalesced. The response produced from each chamber was independent of that produced from other chambers. Damping material was strategically located on the surface of a piezoelectric device to increase vibration decay rate [2]. The damping provided additional enhancement to the audio response.

Following this introduction is a description of a piezoelectric device whose mass and stiffness distribution was tailored to have a vibration response with numerous high-amplitude vibration modes. Mapping the piezoelectric device surface topography during vibration to ascertain locations for stroke-type actuation is presented next. Enhancement of audio response when an acoustic chamber is affixed to the surface of the piezoelectric device follows. The results of vibration response and audio response when damping is applied to the actuator surface are presented next. The audio response from using all the aforementioned techniques is then presented followed by conclusions.

2. Tailoring vibration response

The first technique used to enhance piezoelectric actuator audio output is to develop a vibration response that results in multiple high-amplitude natural resonants which span the 0–1 KHz frequency range. Having the resonants in that range provides a means of increasing audio output in the lower frequency range thereby extending the audio bandwidth. Fig. 1 shows a piezoelectric device that has been developed to have numerous natural frequencies with high-amplitude displacements. Key features of the device are annotated in Fig. 1. The device has a "T" planform (i.e., throat and crossbar). The device can be either of unimorph (a single electrically active piezoceramic layer) or bimorph (two electrically active piezoceramic layers with electric fields out-of phase with respect to each other) construction. The outer metallic layers are a conductive metal such as stainless steel or brass. A conductive epoxy adheres the piezoceramic to the metal layers. Crossbar dimensions for the device in Fig. 1 are $25.4\,\mathrm{mm} \times 44.5\,\mathrm{mm}$. The throat dimensions are $12.7\,\mathrm{mm} \times 12.7\,\mathrm{mm}$.

The device is clamped at its throat with only the crossbar exposed (i.e., cantilevered). The throat has a low torsion and bending stiffness; yet, can sustain the large-amplitude vibration without breaking. The mass distribution about the throat, clamping at the throat and the low stiffness at the throat resulted in a small surface area with numerous high-amplitude natural frequencies below 1 kHz. The device can be dynamically scaled by maintaining the ratio of the inertia about

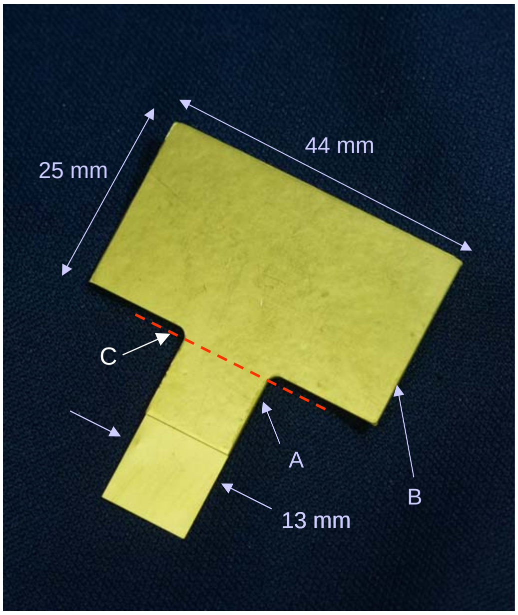


Fig. 1. Piezoelectric device with mounting throat. The annotated features are (A) throat, (B) crossbar, (C) cantilever line.

the clamping line to the throat cross-sectional stiffness. A rectangular or oval actuator planform would require a very large surface area or high aspect ratio to achieve the same low-natural frequencies.

When unimoprh construction is used with an applied voltage of ± 25 V, the bending displacement amplitude at the edge has been measured to be 3 mm with the edge 25.4 mm away from the clamping line. The actuator using unimorph construction has three natural frequencies with amplitudes greater than 0.76 mm. The first bending mode at 111 Hz has an edge displacement of 3 mm. The out-of-plane displacements measured at 743 and 426 Hz exceeded 0.76 mm. Although these amplitude are significantly less than the first bending, their amplitudes are more than sufficient for audio output. The displacement at the 977 Hz natural frequency could be seen with the naked eye using ± 25 V applied voltage. The device presented here can use a simple and very small DC–DC converter to achieve the desired voltage from a 5 V battery.

3. Vibration topography mapping

The second technique to enhance audio output of piezoelectric devices is to increase the effectiveness of harnessing energy from a vibrating piezoelectric actuator. The crossbar deformation shapes for the various natural vibration frequencies of the device are shown in Fig. 2. The dynamic topography that occurs during vibration is measured using a laser vibrometer for a grid of points along the surface of the piezoelectric device. A sine sweep of frequency is used at each grid point. Frequency responses of all grid points are then analytically reduced to deformation shapes. The mapping is used to identify all lines and points of peak out-of-plane

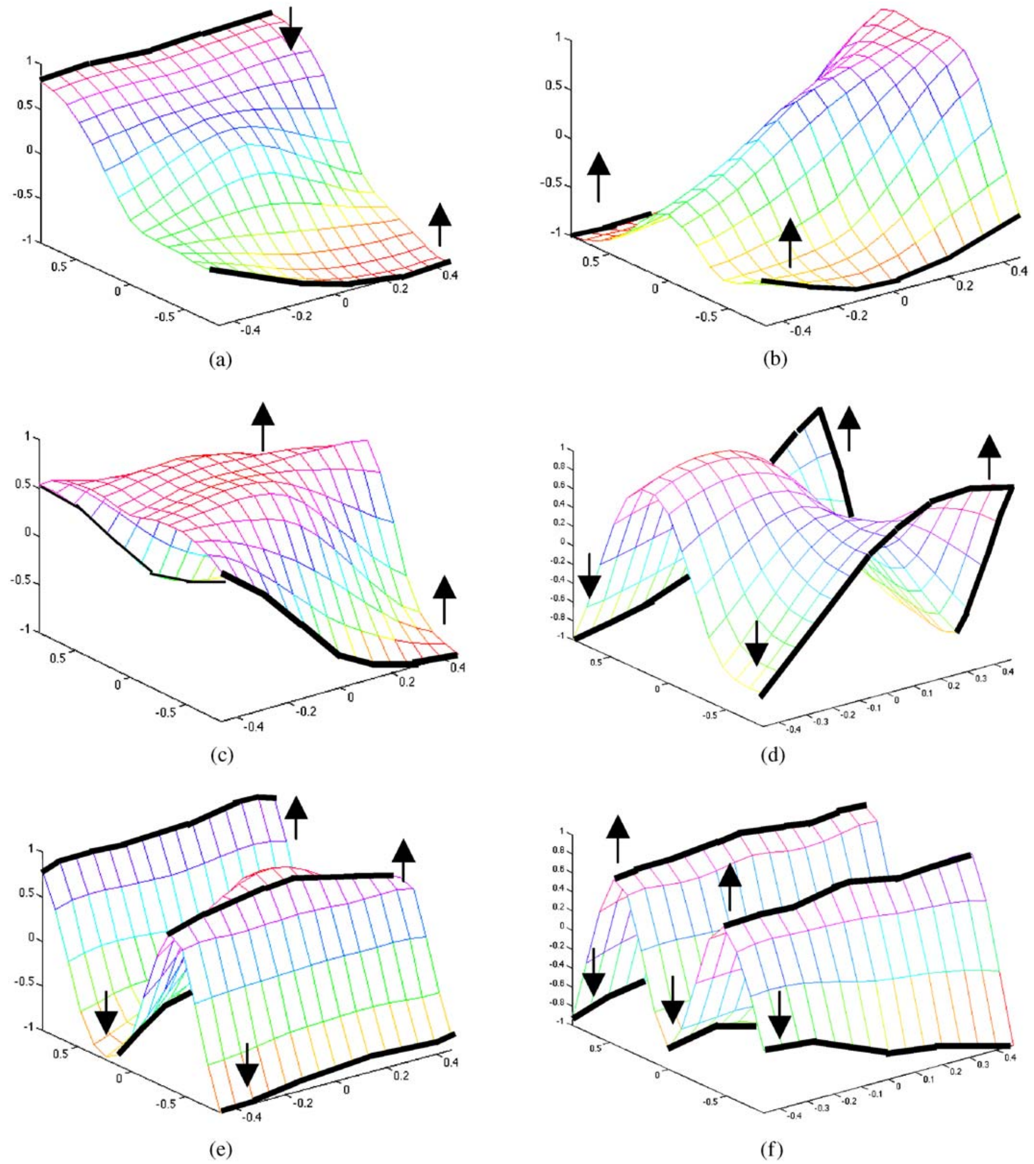


Fig. 2. Vibration deformation shapes of the piezoelectric device crossbar. The lines of maximum out-of-plane displacement are shown in black. The device is cantilevered at its throat. (a) at 176 Hz, (b) 530 Hz, (c) 977 Hz, (d) 1730 Hz, (e), 1898 Hz, (f) 3580 Hz.

displacement suitable for stroke-like actuation. During vibration, the cyclic out-of-plane motion of each displacement line produces reciprocating strokes in a manner similar to a piston [3–4].

Using the mapping technique, the potential actuation positions (black lines) are shown along the peak displacement lines in Fig. 2. The direction of the arrows indicate the relative phase of the displacement lines with respect to neighboring lines. In Fig. 3, all the peak out-of-plane displacement points are mapped to the undeformed device. The mapping is done for all vibration

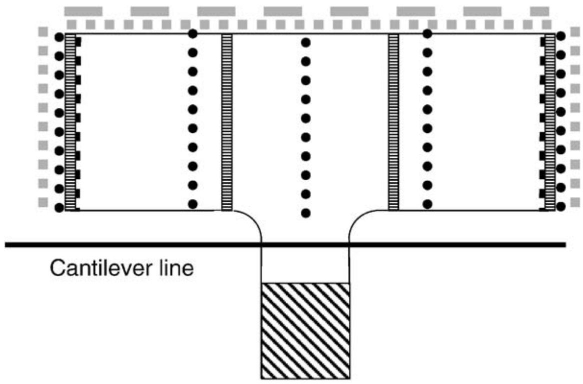


Fig. 3. Mapping of peak out-of-plane displacement lines and points for the piezoelectric actuator. The most effective actuator positions are along these lines. The device is clamped at the throat with the mapped portion of the device exposed. The displacement lines are: 111 Hz, ▬ ▬▬ ▬▬; 176 Hz, ■ ■; 530 Hz, ●●●●; 977 Hz, ▬ ▪ ▬; 1730 Hz, ▪ ▪ ▪ ▪; 1898 Hz, ▥▥▥; 3580 Hz, ● ● ●.

modes within a desired range of operation. The most effective actuator positions are along these lines. Note that at least one peak displacement line for each natural frequency terminates at either the crossbar corners or its centerline. Hence, there are six locations on the device for which the vibration at one or more of the natural frequencies mapped will have the highest out-of-plane displacements. The lines can be used simultaneously or individually. Stroke-like actuation from one peak-out-of-plane line is independent of actuation produced from other lines.

Many of the vibration deformations have amplitudes that are not discernable to the naked eye but can be measured using non-obtrusive methods such as laser vibrometry. Many of these amplitudes are sufficient for acoustic applications. Currently in the design of piezoelectric actuators, only the deformation of the fundamental vibration mode (lowest frequency) produces sufficient out-of-plane displacement for audio devices. Hence, to date, points along the peak displacement lines for the lowest natural frequency would be a logical choice for actuation. The identified locations on the surface are the most suitable for a single actuation point or a collection of simultaneous actuation points. The significance of this method is that a single piezoelectric actuator with the vibration response of the device in Fig. 1 can function simultaneously as a collection of actuators. The next section will discuss how the displacement lines were used to drive a collection of Helmholz-like acoustic chambers affixed to the surface of the device.

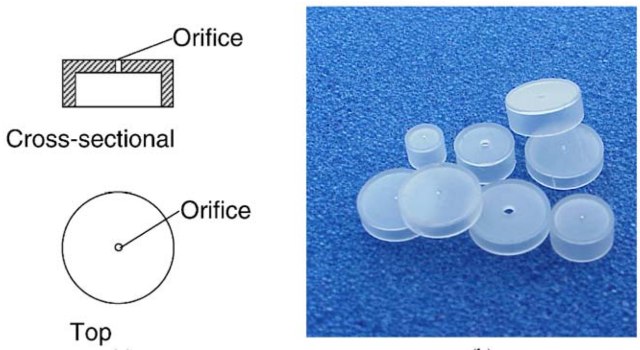
4. Acoustic chamber use

In the third technique, acoustic chambers are affixed to positions along one or more vibration peak out-of-plane displacement lines. The map shown in Fig. 3 is used to identify locations to place the chambers. The best locations are the points where lines from different natural frequencies coalesce; such as, the actuator corners and centerline. Locating multiple chambers on the piezoelectric device surface makes it possible for a single actuator to drive numerous sound sources. When multiple chambers are affixed to the piezoelectric device, they are placed symmetric about the centerline so that the mass symmetry of the device is maintained. The sound produced from each chamber is independent of the other chambers. The sounds are produced as each chamber is oscillated in the air while the piezoelectric actuator vibrates. Typical audio devices use a single driver (e.g., speaker cone driven by magnet within solenoid) to produce a single sound source.

Each acoustic chamber is formed as a cylinder with its bottom surface removed. The top and cross-sectional views of the chamber are shown in Fig. 4(a). The top surface has an orifice. Various sizes of the chambers are shown in Fig. 4(b). Acrylonitrile-butadiene-styrene (ABS) plastic was used to fabricate the chambers. The diameters were either 5, 7.6 or 10.2 mm. Chamber heights were either 2.54 or 3.81 mm. Orifice sizes were either 0.25, 1.27 or 1.91 mm. Chamber geometry is sized to prevent any vibration mode from being constrained. When affixed to the surface of the piezoelectric device (Fig. 1), an acoustic chamber is formed. An elastic double-sided adhesive film is used to attach the chamber to the surface of the piezoelectric device.

Results of using a single acoustic chamber are shown in Fig. 5. Two audio measurements and a vibration response measurement were taken at a point on the device (Fig. 1) located 8 mm from the top edge of the device and 11 mm from the side edge. A Polytec laser vibrometer was used for vibration response measurements. A Bruel and Kjaer 3.2 mm diameter microphone was used for the audio measurements. The small diameter microphone was used to focus on the sound in proximity to the measurement point (i.e., reduce audio response from other portions of the device from being integrated into the measurement). During all audio measurements and during the vibration measurement, ± 25 V was applied to the device.

The two audio measurements consisted of a baseline measurement without the chamber and one with the chamber. A chamber with 5 mm diameter, 3.81 mm height, 0.25 mm orifice diameter and wall thickness of 0.51 mm was used. During the audio measurement, the microphone was placed 2 mm from the surface (without chamber) and 2 mm from the chamber (when chamber is affixed to surface). A test fixture was developed which allowed the device to have both acoustic measurements and vibration measurements taken without removing the specimen from the fixture and therefore resulting in the same support conditions of the piezoelectric device for acoustic and vibration testing. A unimorph device with geometric planform described in the previous section was used for results produced in Fig. 5.



(a) (b)

Fig. 4. Acoustic chamber: (a) acoustic chamber schematic, (b) acoustic chambers.

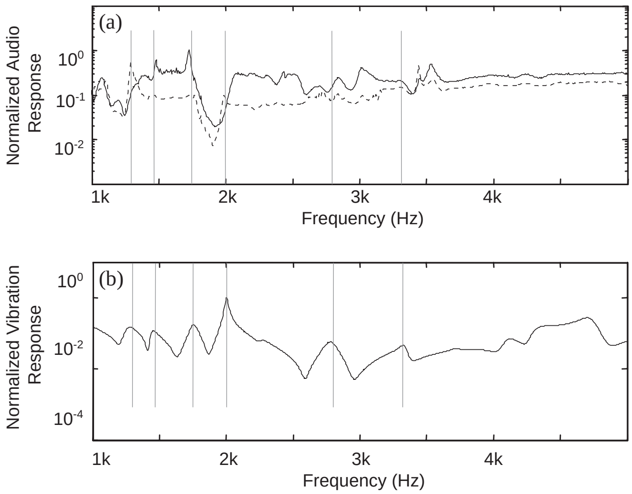


Fig. 5. Audio response (—) using an acoustic chamber affixed to the surface of the piezoelectric actuator, compared to the response (- - -) without the chamber. The audio response is referenced to the vibration response measured at the same point. (a) normalized audio response, (b) normalized vibration response.

The peaks in the vibration response are at the natural frequencies of the actuator without the chambers attached. Many of the peak displacement lines for the actuator vibration coalesce near the measurement point. The vibration response measurements (Fig. 5(b)) were used to correlate audio response. For example, the reduction of audio response at approximately 1900 Hz is resulting from the reduction in vibration amplitude at the same frequencies. The chamber produced an average increase of 68% in dB level in the 1350–1750 Hz frequency range and 74% dB level increase in the 2020–2570 Hz range.

To date, electrical amplifiers have been used to increase audio output by increasing applied power. In contrast, the method described in this section increased audio output without increasing applied power. The acoustic chambers increased audio output without additional power demonstrated another means of harnessing vibration for acoustic response. A subsequent section will show results when multiple acoustic chambers are used and positioned at locations where numerous lines of peak out-of-plane displacements and points coalesce.

5. Tailoring damping distribution

The fourth technique consists of the strategic use of damping material on the piezoelectric device. The damping material eliminates the distortion produced by persistent vibration. The damping material caused the vibration response and resulting audio response to quickly decay after the stimulus is removed.

Piezoelectric devices have only been used as harmonic ringing devices in applications such as smoke detectors, watch alarms or crude audio devices. The shortcoming of these devices is that when a stimulus excites them, they continue to vibrate. When the stimulus is an audio signal, the audio response resulting from one instant of signal distorts the response created from the next instant. To eliminate the distortion, damping material is applied to the actuator surface. Damping material is positioned on the crossbar only. Mass from the damping material increases the bending inertia and the torsional inertia about the cantilever clamp line. The material does not add additional layers to the throat but only to the crossbar and therefore the bending stiffness and the torsional stiffness about the throat are not altered. The result is that the first bending and first torsion frequencies (first two natural frequencies) are lowered. Fig. 6 shows the vibration response

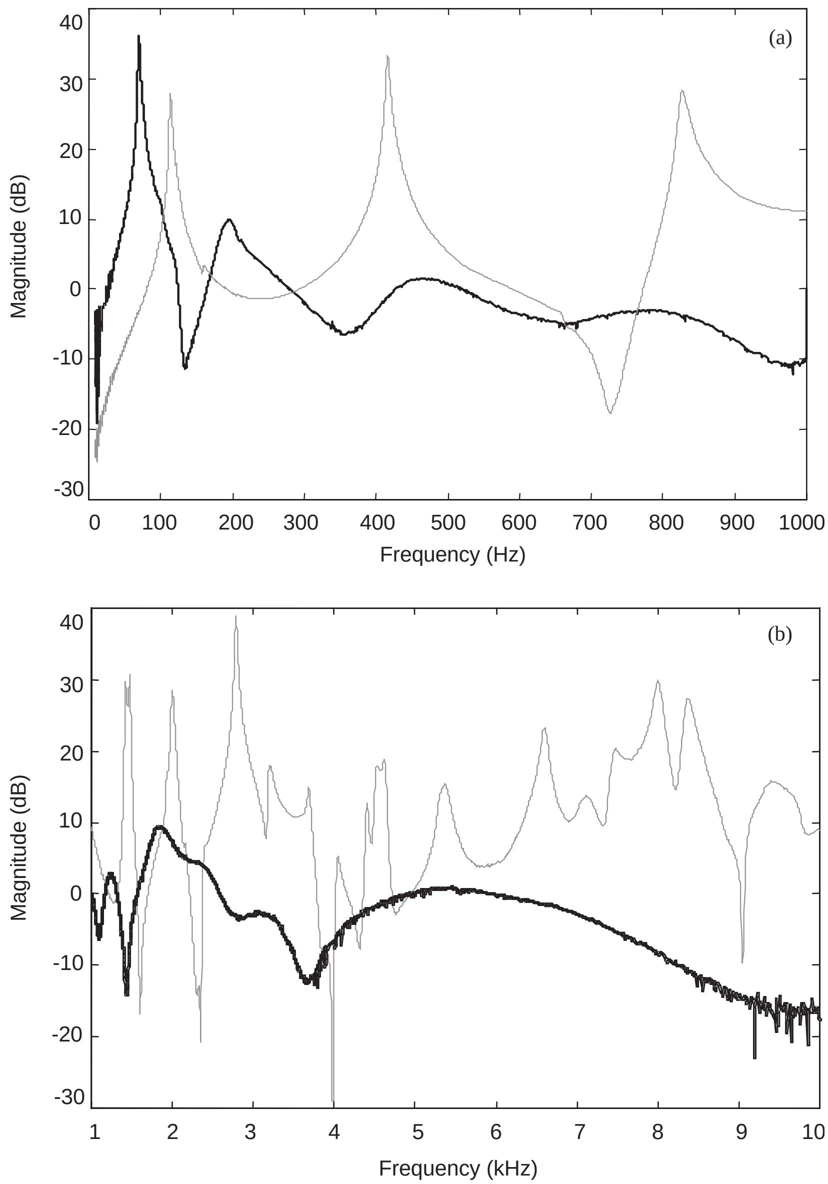


Fig. 6. Vibration response (—) with three layers of synthetic polyolefin (0.76 mm each layer) used for damping compared to the response (—) when no polyolefin is used. Measured at center of cantilever edge. (a) Vibration response 0–1 kHz, (b) vibration response 1–10 kHz.

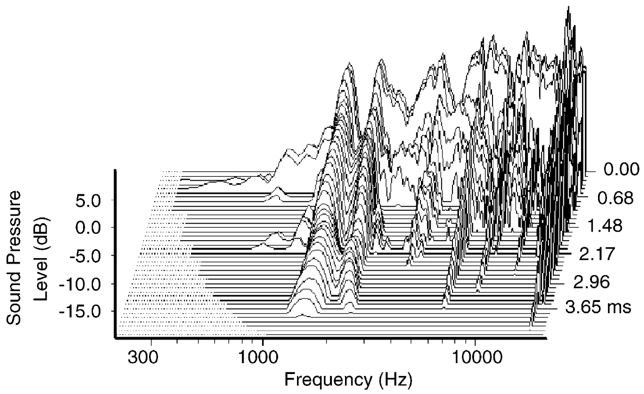


Fig. 7. Sequential spectral plots of piezoelectric audio device response when stimulus is removed. The damping material is six layers of Scotch 468 adhesive.

with three layers of synthetic polyolefin (0.76 mm each layer) compared to the response when no polyolefin is used. Other materials can be used for damping. The response is measured at the center of the actuator's top edge (Fig. 1). The response curves demonstrate that the strategic placement of damping material along the actuator crossbar can be used to attenuate the high-frequency vibration while maintaining sufficient vibration amplitudes for the first two natural frequencies. The damping material further tailors high-frequency vibration response by attenuating resonant peaks. The damping material reduces the resonate peaks at approximately 400 and 850 Hz by 30 dB. Furthermore, the peak-to-trough spread in the range 150–1 kHz is reduced to 20 dB as compared with the initial spread of approximately 55 dB. Similarly, the peak-to-trough spread is reduced from 70 to 25 dB in the 1–10 kHz range. Reduction of the peak-to-trough spread in the vibration response results in the audio response having a lesser peak-to-trough dB spread.

Fig. 7 shows a sequence of frequency responses taken after a stimulus is applied to the audio device (piezoelectric actuator, acoustic chambers and damping material). The complete audio response decays in approximately 3.7 ms. The effect on sound quality is that it produces audio clarity. The damping material such as synthetic polyolefin eliminates excessive vibration without eliminating the flexural characteristics of the actuator.

6. Audio response

This section presents results of using all audio enhancement methods presented in the previous sections. The results are presented for a bimorph (two active layers) construction of the actuator shown in Fig. 1. A schematic of the audio device is shown in Fig. 8. The device is shown in Fig. 9. The audio device consisted of the actuator with the additions of the acoustic chambers affixed to the actuator top surface using one layer of Scotch 468 double-sided adhesive. The Scotch 468 adhesive also functioned as a damping layer on the top surface. Six layers of the Scotch 468 adhesive applied to the bottom layer functioned as damping material. Six acoustic chambers are used with the device. They are positioned at the four corners of the device and at the center of the

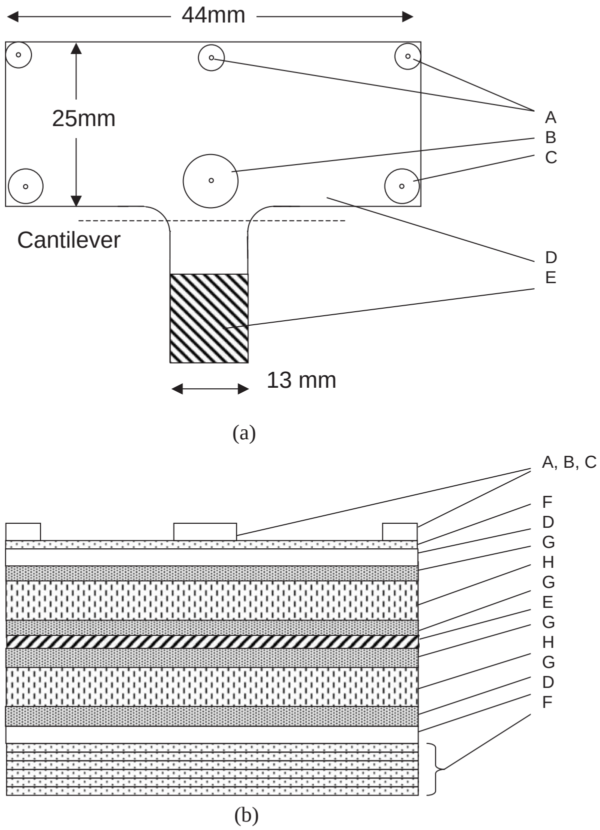


Fig. 8. Schematic of audio device consisting of piezoelectric actuator with acoustic interface (damping layers and six acoustic chambers). The constituent parts are (A) 5 mm diameter acoustic chambers, (B) 10 mm diameter acoustic chambers, (C) 7.6 mm diameter acoustic chamber, (D) brass layer (0.025 mm), (E) brass layer (0.076 mm), (F) damping/adhesive layer, (G) electrically conductive layer and (H) piezoceramic layer (0.20 mm). All chambers used are 2.54 mm in height with 0.25 mm orifice. (a) Top view of audio device, (b) cross-sectional view of audio device.

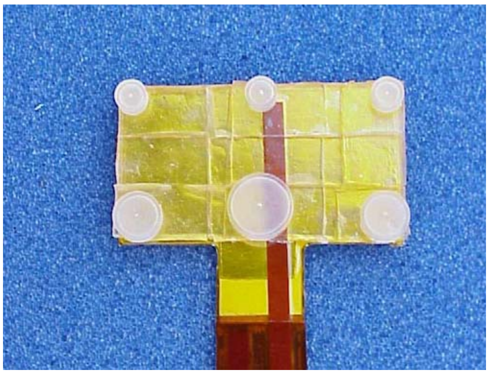


Fig. 9. Piezoelectric actuator with acoustic interface (damping layers and six acoustic chambers).

crossbar. As can be seen from Fig. 3, the chamber locations are where the peak out-of-plane displacement lines coalesce. The device is cantilevered at the throat along the line shown in Fig. 8(a). All chambers used are 2.54 mm in height with 0.25 mm orifice. All chambers along the top edge have a 5 mm diameter. Chambers in the lower corners have 7.6 mm diameters. The chamber at the lower center has a 10.2 mm diameter.

Fig. 10 shows the impedance for an actuator when two active piezoceramic layers (bimorph) are used in the device. The impedance variation with frequency is similar to that of a capacitor. At 100 Hz the impedance is 3500 Ω resulting in the power used being 7.14 mW when 5 V are applied. The impedance level reduces to approximately 83 Ω at 5 kHz (301 mW), 62 Ω (403 mW) at 7 kHz and 43 Ω (581 mW) at 10 kHz.

The device shown in Fig. 9 has resulted in a piezoelectric acoustic device with the audio response shown in Figs. 11 and 12. Responses shown in Figs. 11 and 12 used ± 14 V. The measurements are taken without any acoustic enclosures. At 1 cm (sufficient distance for headset use), sound pressure level was measured at 67 dB near 0 Hz. The response increased to 88 dB at 600 Hz. From 600 to 5000 Hz, the response is 93 ± 5 dB (± 4 dB for the response at more than 1 kHz). Fig. 12 compares the device with a commercially available Mylar cone of slightly higher cost and more complicated construction. Mylar cone speakers are the most common audio devices used in many portable consumer electronics. As shown in Fig. 12, the piezoelectric device produces higher fidelity from 500 to 5000 Hz than the Mylar cone. The piezoelectric sound is within a 13 dB band as compared to the 16 dB band of the Mylar cone. The piezoelectric device also outperforms the Mylar cones in the 0–500 Hz region. Near 0 Hz, the Mylar cone has 50 dB as compared with 60 dB using the piezoelectric device. At 4500 Hz, the Mylar cone response drops; whereas, the piezoelectric continued to maintain the same approximate level. The Mylar speaker used for comparison had 1.1 V applied with 8 Ω impedance (151 mW). Mylar cone dimensions were 23 mm in diameter and with a height of 8.7 mm. During testing, the piezo audio used 56 mW at 100 Hz and 2360 mW at 5 KHz.

The construction of mylar cone devices is much more complicated than that of the device presented in this paper whose construction consisted of laminating layers together. Many portable consumer electronics can house the device presented in Fig. 9. In Fig. 13, it can be seen that the device is easily integrated into stereo headsets, or as free-standing speakers or cellular phones. The

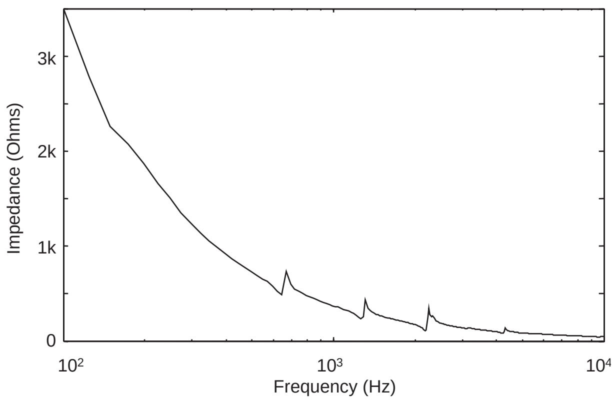


Fig. 10. Electrical impedance when two active piezoceramic layers are used in the audio device.

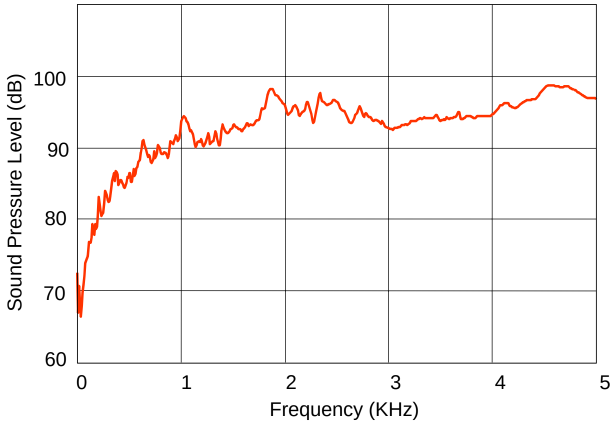


Fig. 11. Piezoelectric audio device sound pressure level measured at 1 cm without any sound enclosure.

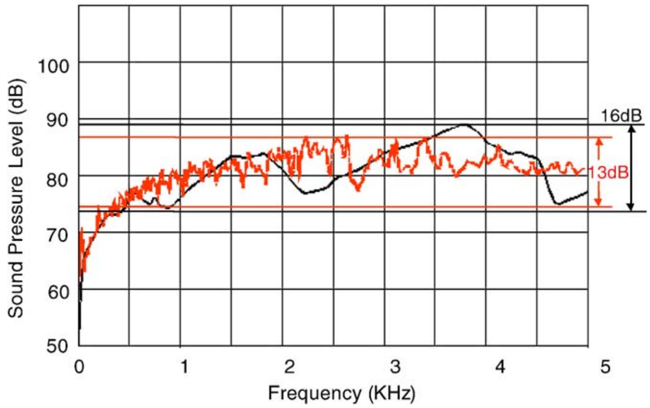
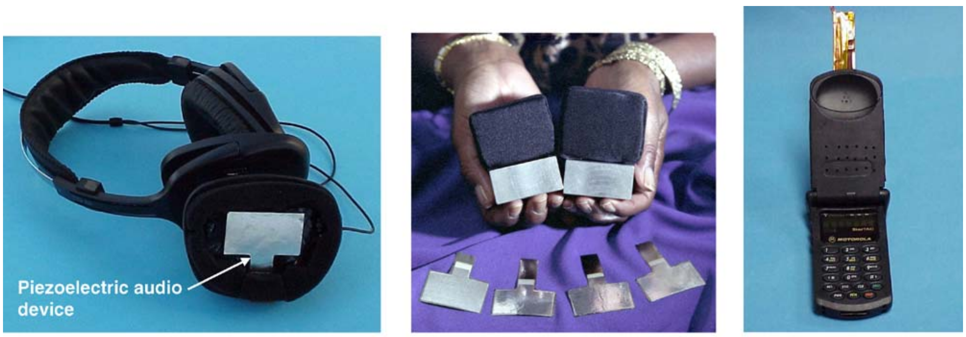


Fig. 12. Piezoelectric device audio response (–) measured without any acoustic enclosure, compared with commercially available electromagnetically driven Mylar cone response (—) mounted within an acoustic enclosure. All sound pressure levels measured from 10 cm.



(a) (b) (c)

Fig. 13. Application of the piezoelectric audio devices: (a) mounted in headphones, (b) used as speakers, (c) mounted inside a cell phone.

simplicity of the design lends itself to lower cost. The piezoelectric device can be used without its acoustic chambers and thereby resulting in a very low profile of 0.6 mm thickness. The electrical leads can be soldered on or are part of the electronic system on which the device will be used (i.e., pressure contact). With the acoustic chambers attached, the profile is thinner than most Mylar cones of comparable performance. Another feature of the device presented in this paper that gives it an advantage over mylar cones is that the piezoelectric device weighs substantially less (4 g) than the mylar cones which use magnets (10 g).

7. Concluding remarks

Four methods to increase sound fidelity and quality produced from piezoelectric actuators have been presented. The first method developed a prescribed vibration response by tailoring physical attributes of the piezoelectric actuator. The second method consisted of mapping the vibration deformation topography of the piezoelectric device for all natural resonances to determine the most effective locations for force actuation within the frequency range of interest. The identified locations on the surface were the most suitable to strategically position a single actuation point or a collection of simultaneous actuation points. The tailored attributes included mass distribution, stiffness distribution, damping distribution and boundary conditions. A "T" planform actuator was produced. The mass distribution about the throat; cantilever at the throat; and, the low torsion and low bending stiffness at the throat resulted in numerous high-amplitude natural frequencies. The actuator had numerous low natural frequencies for a small surface area. When an unimoprh construction was used, the bending displacement amplitude at the edge was measured to be 3 mm with the edge 25.4 mm away from the cantilever line using an applied voltage of amplitude of ± 25 V. The actuator using unimorph construction had three natural frequencies with amplitude greater than 0.76 mm. The first bending mode had an edge displacement of 3 mm. The out-of-plane displacements measured at 743 and 426 Hz exceeded 0.76 mm. The 977 Hz natural frequency could be seen with the naked eye using ± 25 V applied.

The third method coupled the piezoelectric device to a collection of acoustic chambers. Results demonstrated that these chambers increase sound pressure level without additional applied power. To date, electrical amplifiers have been used to increase audio output by increasing applied electrical power. The chambers were located at the locations upon the piezoelectric device surface where the vibration deformation lines of peak out-of-plane displacement coalesced. The sound produced from each chamber was independent of that produced from another chamber. Hence, the single piezoelectric device produced multiple sound sources (compared with the single sound source produce from a voice coil actuator and speaker cone). This method demonstrated that a single piezoelectric actuator can function simultaneously as a collection of actuators. The fourth method of developing the audio response was to strategically locate damping material on the surface of a piezoelectric device to increase vibration decay rate.

The methods were applied to piezoelectric devices. The piezoelectric actuators serving as the acoustic drivers were less than 1 mm in thickness. Six acoustic chambers were affixed to the piezoelectric device surface. At 1 cm (sufficient distance for headset use), sound pressure level was measured at 67 dB near 0 Hz with applied voltage of ± 14 V. The response increased to 88 dB at 600 Hz. From 600 to 5000 Hz, the response is 93 ± 5 dB (± 4 dB for response greater than 1 kHz).

The piezoelectric device produced higher fidelity from 500 to 5000 Hz than a commercially available Mylar cone. The piezoelectric sound is within a 13 dB band as compared to the 16 dB band of the Mylar cone. The piezoelectric device also outperforms the Mylar cones in the 0–500 Hz region. Near 0 Hz, the Mylar cone has 50 dB as compared with 60 dB using the piezoelectric device.

The application of the methods presented in this monograph resulted in high fidelity, high-bandwidth, and high sound-quality audio devices with low profiles. The methods are also applicable to other vibrating lamina such as externally excited plates and films. The small size (exposed area 25.4 mm $\times$ 44.5 mm) of the piezoelectric acoustic device makes it applicable to many consumer electronic products such as pagers, portable radios, head phones, stereo headsets, laptop computers, cellular phones, computer monitors, toys, electronic games or as free-standing speakers. The devices presented in this monograph have extreme fabrication simplicity. The entire piezoelectric audio device was fabricated by lamination.

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